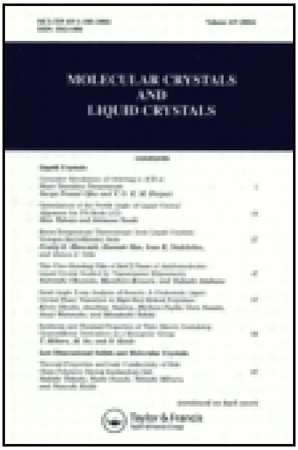

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Propagation and Scattering of Light in Helical Liquid Crystals with Large Pitch

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The problems of light propagation and scattering in twisted nematic and cholesteric liquid crystals are considered. We study the correlation function of the director fluctuations, the eigenwaves, and the Green's function for helical liquid crystals with the pitch, which significantly exceeds the wavelength of light. The obtained results make possible to describe angular and polarization dependencies in scattered light, the wave-guide channel formation, the turning points and the effect of percolation through the forbidden zone.

Keywords: helical liquid crystal; scattering of light; spatial correlation function; wave-guide channel

1. INTRODUCTION

Light scattering technique is one of the effective methods for studying of the liquid crystal (LC) systems. Scattering of light is usually considered under the assumption that the medium is spatially homogeneous, or fluctuations and the propagation of waves in inhomogeneous media are described on the basis of small parameters. The problem becomes more complicated when the spatial homogeneity of the medium is violated considerably. This leads to several problems such as the description of the structure of the normal waves, the calculation of the Green's function of the electromagnetic field, and analysis of the correlation function of permittivity fluctuations. Typical examples are the problems of propagation and scattering of light in media

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with periodically varying properties. Such media include cholesteric liquid crystals (CLC), twisted nematic liquid crystals, and some kinds of smectic liquid crystals.

The exact solution of the problem of electromagnetic waves propagation in CLC was obtained only for waves propagating along the symmetry axis of the system [1–3]. The formal analytic solution for the problem in the case of oblique incidence [4,5] has the form of an infinite series and is difficult for an analysis. Therefore, various approximate methods are widely used in the optics of layered liquid crystals [6,7]. The main attention is paid to the situation when the wavelength is of the order of the pitch. In this case methods developed in X-ray diffraction theory are effective [7].

Such an approach remains most popular for CLC up to now [8–11]. The problems with the normal waves and the field of a point source [12] as well as the spectrum of the thermal fluctuations of the director [13,14] have been investigated using this method.

The opposite case, when the wavelength is much less than the pitch, has been studied insufficiently. However, this problem has become important in recent years in connection with application of nematic twist cells and CLC with a large pitch in display systems [15].

If light propagates along the axis of a helix, the adiabatic regime takes place, then the polarization of waves rotates together with the optical axis. In the general case of oblique incidence for systems with a large pitch it is natural to use the WKB (Wentzel-Kramers-Brillouin) method since the size of inhomogeneities is much larger than the wavelength. It is difficult to apply the WKB method for electromagnetic waves directly since this gives rise to a system of coupled equations. This problem was solved in [16,17] for electromagnetic waves propagating in locally isotropic media with smooth inhomogeneities. For CLC with the large pitch the generalization of the WKB method was suggested in [18].

In this work we developed a general scheme of calculation of light scattering intensity in CLC with the pitch significantly exceeding the wavelength. The approach based on the Kirchhoff method provides explicit expressions for angular and polarization dependencies of single light scattering intensity.

2. CORRELATION FUNCTION OF THE DIRECTOR FLUCTUATIONS

The elastic free energy of the cholesteric liquid crystal has the form [19]

$$F = \frac{1}{2}\int d\mathbf{r}[K_{11}(\mathrm{div}\,\mathbf{n})^2 + \mathrm{K}_{22}(\mathbf{n}\cdot\mathrm{curl}\,\mathbf{n} + \mathrm{q}_0)^2 + \mathrm{K}_{33}(\mathbf{n}\times\mathrm{curl}\,\mathbf{n})^2], \quad (2.1)$$

where $K_{ll}(l=1,2,3)$ are the Frank modules. Here we consider the situation when the CLC is unbounded or the surface energy is negligible compared to the volume part of the energy. The unit vector director $\mathbf{n}=\mathbf{n}(\mathbf{r})$ describes the local orientation of the long axes of molecules. In the equilibrium the energy (2.1) is minimal for helical distribution of the director

$$\mathbf{n}^0(\mathbf{r}) \equiv \mathbf{n}^0(z) = (\cos\phi, \sin\phi, 0). \tag{2.2}$$

Here we introduced the frame with the z axis directed along the CLC axis, $\phi=\phi(z)=q_0 z+\phi_0$, the angle ϕ_0 determines the orientation of the director in the plane $z=0$, $q_0=2\pi/P$, P is the pitch.

The spatial correlation function of the director fluctuations is determined as

$$g_{\alpha\beta}(\mathbf{r}_{1\perp}-\mathbf{r}_{2\perp};z_1,z_2) = \langle \delta n_\alpha(\mathbf{r}_{1\perp},z_1)\delta n_\beta(\mathbf{r}_{2\perp},z_2)\rangle, \tag{2.3}$$

where

$$\delta\mathbf{n}(\mathbf{r}) = \mathbf{n}(\mathbf{r}) - \mathbf{n}^0(z) \tag{2.4}$$

and the brackets $\langle\ldots\rangle$ designate the statistical averaging.

In order to calculate the correlation function of the director fluctuations in the Gaussian approximation we restrict ourself to the quadratic terms over $\delta\mathbf{n}$ in the free energy (2.1)

$$\delta F = \frac{1}{2}\int d\mathbf{r}\{K_{11}(\nabla\cdot\delta\mathbf{n})^2 + K_{22}[\mathbf{n}^0\cdot(\nabla\times\delta\mathbf{n})]^2 + K_{33}[(\delta\mathbf{n}\cdot\nabla)\mathbf{n}^0 + (\mathbf{n}^0\cdot\nabla)\delta\mathbf{n}]^2\}. \tag{2.5}$$

Here we take into account the relations $\operatorname{div}\mathbf{n}^0=0$ and curl $\mathbf{n}^0=-q_0\mathbf{n}^0$ which are valid for the helical structure (2.2). As far as $|\mathbf{n}|=|\mathbf{n}^0|=1$ the condition $\delta\mathbf{n}\perp\mathbf{n}^0$ is valid in the first order in δn. Therefore $\delta\mathbf{n}$ can be parameterized by two functions [13,14]:

$$\begin{aligned}\delta n_x(\mathbf{r}) &= -u_1(\mathbf{r})\sin\phi(z),\\ \delta n_y(\mathbf{r}) &= u_1(\mathbf{r})\cos\phi(z),\\ \delta n_z(\mathbf{r}) &= u_2(\mathbf{r}).\end{aligned} \tag{2.6}$$

The modes u_1 and u_2 determine the director fluctuations in the xy plane and along the z axis respectively. In vector notations we can write

$$\delta\mathbf{n}(\mathbf{r}) = u_1(\mathbf{r})\mathbf{h}^{(1)}(z) + u_2(\mathbf{r})\mathbf{h}^{(2)}, \tag{2.7}$$

where

$$\mathbf{h}^{(1)}(z) = \mathbf{h}^{(2)} \times \mathbf{n}^0(z), \quad \mathbf{h}^{(2)} = \mathbf{e}_z \tag{2.8}$$

and $\mathbf{e}_z$ is the unit vector directed along the z axis.

From Eq. (2.7) we can express the correlation function of the director fluctuations through the correlation matrix of the scalar functions $u_{1,2}$

$$g_{\alpha\beta}(\mathbf{r}_\perp; z_1, z_2) = \sum_{k,l=1}^{2} G_{kl}(\mathbf{r}_\perp; z_1, z_2) h_\alpha^{(k)}(z_1) h_\beta^{(l)}(z_2), \tag{2.9}$$

where

$$G_{kl}(\mathbf{r}_{1\perp} - \mathbf{r}_{2\perp}; z_1, z_2) \equiv G_{kl}(\mathbf{r}_1, \mathbf{r}_2) = \langle u_k(\mathbf{r}_1) u_l(\mathbf{r}_2) \rangle. \tag{2.10}$$

As far as in equilibrium CLC is spatially homogeneous in the plane normal to the z axis we use a two-dimensional Fourier transformation. Substituting Eq. (2.6) into Eq. (2.5) and completing two-dimensional Fourier transformation we can get the distortion energy in the form

$$\delta F = \int \frac{d^2 q}{(2\pi)^2} \delta F_{\mathbf{q}}. \tag{2.11}$$

Integrating by parts and omitting the terms outside the integral we present the value $\delta F_{\mathbf{q}}$ as a quadratic form

$$\delta F_{\mathbf{q}} = \frac{1}{2} \int \mathbf{u}^T(\mathbf{q}, z) \widehat{\mathcal{A}}(\mathbf{q}, z) \mathbf{u}(\mathbf{q}, z) dz \tag{2.12}$$

with

$$\mathbf{u} = (u_1, u_2)^T.$$

The matrix $\widehat{\mathcal{A}}$ is a differential operator of the second order. In the coordinate frame with the x axis directed along the $\mathbf{q}$ vector ($q_x = q$, $q_y = 0$) it has the form

$$\widehat{\mathcal{A}} = K_{11} \begin{pmatrix} q^2 \sin^2\phi & iq \sin\phi \partial_z \\ iq\partial_z \sin\phi & -\partial_z^2 \end{pmatrix} + K_{22} \begin{pmatrix} -\partial_z^2 & -iq\partial_z \sin\phi \\ -iq \sin\phi \partial_z & q^2 \sin^2\phi \end{pmatrix}$$
$$+ K_{33} \begin{pmatrix} q^2 \cos^2\phi & -iq_0 q \cos\phi \\ iq_0 q \cos\phi & q^2 \cos^2\phi + q_0^2 \end{pmatrix}, \tag{2.13}$$

where $\partial_z \equiv \partial/\partial z$, $\partial_z^2 \equiv \partial^2/\partial z^2$.

The probability of fluctuations is proportional to $\exp[-\delta F_{\mathbf{q}}/k_B T]$ where k_B is the Boltzmann constant and T is a temperature. The calculation of the correlation function leads to inversion of the $\widehat{\mathcal{A}}$ matrix [20]. This procedure is equivalent to solution of the equation

$$\widehat{\mathcal{A}}(\mathbf{q},z)\widehat{G}(\mathbf{q};z,z_1) = k_B T \delta(z-z_1)\widehat{I}. \tag{2.14}$$

For unambiguous solution Eq. (2.14) has to be complemented by boundary conditions. In the infinite system the principle of the correlations decay, $\widehat{G}(\mathbf{q};z,z_1) \to \hat{0}$ for $z \to \pm\infty$, should be used as such conditions.

The correlation function of the director fluctuations $\widehat{G}(z,z_1)$ in CLC with the large pitch was considered in detail in [21]. The matrix $\widehat{G}$ obeys the inhomogeneous system (2.14) of two differential equations with periodic coefficients and decay condition at $z \to \pm\infty$. Note that for $z \neq z_1$ Eq. (2.14) is homogeneous. Primarily we solve the homogeneous equations for $z > z_1$ and $z < z_1$ and then we construct the correlation function using the continuity condition for $\widehat{G}$ and the jump of its derivative for $z = z_1$. The homogeneous equations can be solved by using the vector analog of the WKB approximation. We finally have for the $\widehat{G}$ matrix

$$\widehat{G}(\mathbf{q};z_1,z_2) = \widehat{G}^{(1)}(\mathbf{q};z_1,z_2) + \widehat{G}^{(2)}(\mathbf{q};z_1,z_2), \tag{2.15}$$

where

$$G_{kl}^{(j)}(\mathbf{q};z_1,z_2) = \frac{k_B T}{2qK_{33}\cos\phi(z_1)\cos\phi(z_2)} \exp\left(-q\left|\int_{z_1}^{z_2} \mu_j(z)dz\right|\right) \times \ell_k^{(j)}(\mathbf{q};z_1,z_2)\ell_l^{(j)*}(\mathbf{q};z_2,z_1), \tag{2.16}$$

where

$$\mu_l(z) = \sqrt{\sin^2\phi(z) + \frac{K_{33}}{K_{ll}}\cos^2\phi(z)}, \quad l = 1,2 \tag{2.17}$$

$$\begin{aligned} \ell^{(1)}(\mathbf{q};z,z') &= \left(i\,\mathrm{sign}(z-z')\frac{\sin\phi(z)}{\sqrt{\mu_1(z)}}, \sqrt{\mu_1(z)}\right), \\ \ell^{(2)}(\mathbf{q};z,z') &= \left(\sqrt{\mu_2(z)},\ i\,\mathrm{sign}(z'-z)\frac{\sin\phi(z)}{\sqrt{\mu_2(z)}}\right). \end{aligned} \tag{2.18}$$

Summing over k and l in Eq. (2.9) and using Eqs. (2.8), (2.15)–(2.18) we get the correlation function of the director fluctuations in the form

$$g_{\alpha\beta}(\mathbf{q};z_1,z_2) = \frac{k_B T}{2qK_{33}\cos\phi(z_1)\cos\phi(z_2)} \sum_{j=1}^{2} \exp\left(-q\left|\int_{z_1}^{z_2} \mu_j(z)dz\right|\right) \times f_\alpha^{(j)}(\mathbf{q};z_1,z_1-z_2) f_\beta^{(j)*}(\mathbf{q};z_2,z_2-z_1), \quad (2.19)$$

where $\mathbf{f}^{(j)}(\mathbf{q};z,z-z') = \sum_{k=1,2} \ell_k^{(j)}(\mathbf{q};z,z')\mathbf{h}^{(k)}(z)$.

Note that the correlation function (2.19) grows in points where $\cos\phi(z_{1,2})$ in the denominator of Eq. (2.16) tends to zero. These points occur in the region where the WKB approximation is violated.

The range of applicability of the WKB approximation is determined by the inequality

$$\frac{q}{q_0}\mu_l \gg 1. \quad (2.20)$$

The correlation function behavior in the vicinity of points where inequality (2.20) is not fulfilled is based on methods which are used for investigation of the turning points in the WKB method. We have discussed this problem in detail in [21]. It is shown there that the correlation function is finite in the points with $\cos\phi(z_1) = 0$ and $\cos\phi(z_2) = 0$.

3. THE NORMAL WAVES

The permittivity tensor $\hat{\varepsilon}$ describes the optical properties of cholesterics. For CLC in equilibrium it has the form [19]

$$\varepsilon^0_{\alpha\beta}(\mathbf{r}) \equiv \varepsilon^0_{\alpha\beta}(z) = \varepsilon_\perp \delta_{\alpha\beta} + \varepsilon_a n^0_\alpha(z) n^0_\beta(z), \quad (3.1)$$

where $\varepsilon_a = \varepsilon_\parallel - \varepsilon_\perp$, $\varepsilon_\parallel$, $\varepsilon_\perp$ are the permittivities along and perpendicular to $\mathbf{n}^0$ respectively. In what follows we consider $\varepsilon_a > 0$. In the general case $\hat{\varepsilon}(\mathbf{r}) = \hat{\varepsilon}^0(\mathbf{r}) + \delta\hat{\varepsilon}(\mathbf{r})$ where $\delta\hat{\varepsilon}(\mathbf{r})$ is the fluctuation of the permittivity tensor.

The wave equation in a non-magnetic medium for a monochromatic wave is:

$$(\text{curl curl} - k_0^2\hat{\varepsilon}(\mathbf{r}))\mathbf{E}(\mathbf{r}) = 0, \quad (3.2)$$

where $\mathbf{E}$ is electric field, $k_0 = \omega/c, \omega$ is the circular frequency, c is the light velocity in vacuum.

Let us solve Eq. (3.2) for equilibrium CLC with the large pitch, $\lambda \ll P$, where λ is the wavelength of light, so we have a large parameter Ω

$$\Omega = k_0/q_0 = P/\lambda \gg 1.$$

In this case it is reasonable to suppose that in each fixed point the electric field has the form of a quasiplane wave

$$\mathbf{E}(\mathbf{r}) = \mathbf{A}(\mathbf{r}) \exp(i\Psi(\mathbf{r})), \tag{3.3}$$

where $\Psi(\mathbf{r})$ is the real phase, $\mathbf{A}(\mathbf{r}) = A(\mathbf{r})\mathbf{e}(\mathbf{r})$, $\mathbf{e}(\mathbf{r})$ is the unit vector of polarization, $\mathbf{e} \cdot \mathbf{e}^* = 1$, and $A(\mathbf{r})$ is the real amplitude. Substituting Eq. (3.3) into the wave equation (3.2) we get

$$\begin{aligned} \nabla \times \nabla \times \mathbf{A}(\mathbf{r}) + i[\mathbf{k}(\mathbf{r}) \times \nabla \times \mathbf{A}(\mathbf{r}) + \nabla \times \mathbf{k}(\mathbf{r}) \times \mathbf{A}(\mathbf{r})] \\ -\mathbf{k}(\mathbf{r}) \times \mathbf{k}(\mathbf{r}) \times \mathbf{A}(\mathbf{r}) - k_0^2 \hat{\varepsilon}^0(\mathbf{r})\mathbf{A}(\mathbf{r}) = 0, \end{aligned} \tag{3.4}$$

where the three-dimensional wave vector $\mathbf{k}(\mathbf{r}) = \nabla\Psi(\mathbf{r})$ is introduced.

In comparison to the first term the second and the third terms are of the order of Ω, the fourth and the fifth terms are of the order of Ω^2. This hierarchy makes it possible to use the geometric optics approximation. If we keep the principal terms only ($\sim\Omega^2$) then we get the vector analog of the eikonal equation. In this approximation we can calculate $\nabla\Psi = \mathbf{k}$ and the polarization vector $\mathbf{e}$. The terms of the order of Ω yield the so-called transfer equation. This equation makes possible to determine the wave amplitude $A(\mathbf{r})$.

According to Eq. (3.4) the eikonal equation has the form

$$\mathbf{k}(\mathbf{r}) \times \mathbf{k}(\mathbf{r}) \times \mathbf{e}(\mathbf{r}) + k_0^2 \hat{\varepsilon}^0(\mathbf{r})\mathbf{e}(\mathbf{r}) = 0. \tag{3.5}$$

For each fixed point $\mathbf{r}$ Eq. (3.5) coincides formally with the equation describing the propagation of plane waves in homogeneous anisotropic media [22], so that it is possible to use well known results. Note that in our case $\hat{\varepsilon}^0(\mathbf{r}) = \hat{\varepsilon}^0(z)$ and the medium is homogeneous in the (x,y) plane. Then $\mathbf{k}(\mathbf{r}) = \mathbf{k}(z) = (\mathbf{q}, k_z(z))$.

Equation (3.5) for uniaxial homogeneous media for the fixed $\mathbf{k}$ direction has two well known solutions corresponding to ordinary and extraordinary waves. The module of the wave vector $\mathbf{k}$ has the form

$$\begin{aligned} k^{(1)} &= k_0\sqrt{\varepsilon_\perp} = k_0 n_{(1)}, \\ k^{(2)}(z) &= k_0\sqrt{\frac{\varepsilon_\perp \varepsilon_\|}{\varepsilon_\perp + \varepsilon_a \cos^2\theta}} = k_0 n_{(2)}(z), \end{aligned} \tag{3.6}$$

where θ is the angle between $\mathbf{n}^0(z)$ and $\mathbf{k}^{(2)}(z)$, $n_{(1)}$ and $n_{(2)}(z)$ are refractive indices of the ordinary and extraordinary waves respectively. In Eq. (3.5) polarization vectors $\mathbf{e}^{(1)}(z)$ and $\mathbf{e}^{(2)}(z)$ corresponding to these values of the $\mathbf{k}$ vectors are determined by conditions

$$\begin{aligned}\mathbf{e}^{(1)}(z) &\parallel \mathbf{k}\times\mathbf{n}^0\\ \mathbf{e}^{(2)}(z) &\parallel \mathbf{n}^0(\mathbf{k}\hat{\varepsilon}^0\mathbf{k})-\mathbf{k}(\mathbf{k}\hat{\varepsilon}^0\mathbf{n}^0).\end{aligned}\tag{3.7}$$

Taking into account that $\cos\theta=\mathbf{k}^{(2)}(z)\cdot\mathbf{n}^0(z)/k^{(2)}(z)=\mathbf{q}\cdot\mathbf{n}^0(z)/k^{(2)}(\mathbf{q},z)$ one can see that the second expression of Eq. (3.6) for given $\mathbf{q}$ becomes an algebraic equation with respect to $k_z^{(2)}(\mathbf{q},\ z)$. Thus we have

$$\begin{aligned}k_z^{(1)}(\mathbf{q},\ z)&\equiv k_z^{(1)}(q)=\sqrt{\varepsilon_\perp k_0^2-q^2},\\ k_z^{(2)}(\mathbf{q},\ z)&=\sqrt{\varepsilon_\parallel k_0^2-q^2-\frac{\varepsilon_a}{\varepsilon_\perp}(\mathbf{q}\cdot\mathbf{n}^0(z))^2}.\end{aligned}\tag{3.8}$$

So we get four solutions for the wave vector $\mathbf{k}_\pm^{(j)}(\mathbf{q},\ z)=(\mathbf{q},\pm k_z^{(j)}(\mathbf{q},\ z))$. The signs "$+$" and "$-$" correspond to waves propagating in the positive and the negative z-direction respectively. Figure 1 shows schematically two solutions (3.6) for a fixed $\mathbf{q}$ vector.

Thus we get four normal waves in the form

$$\mathbf{E}_\pm^{(j)}(\mathbf{r})=A^{(j)}(\mathbf{q};z,\ z_0)\mathbf{e}_\pm^{(j)}(\mathbf{q},\ z)\exp\left(i\mathbf{q}\cdot\mathbf{r}_\perp\pm i\int_{z_0}^{z}k_z^{(j)}(\mathbf{q},\ z')dz'\right)\tag{3.9}$$

The amplitudes $A^{(j)}(\mathbf{q};z,\ z_0)$ will be determined below.

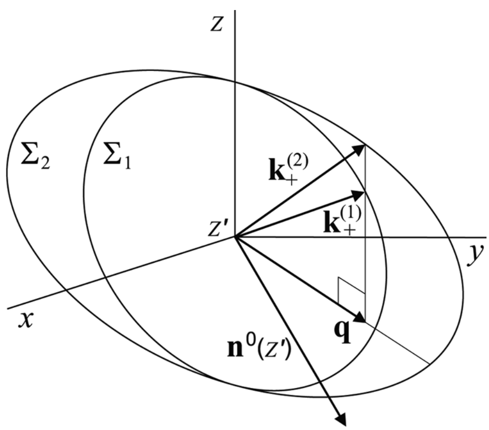


FIGURE 1 Two wave vectors $\mathbf{k}_+^{(1)}$ and $\mathbf{k}_+^{(2)}$ corresponding to given $\mathbf{q}$. Here circle Σ_1 and ellipse Σ_2 are the cross sections of the surfaces of ordinary and extraordinary wave vectors by the plane containing the $\mathbf{q}$ vector and the z axis.

From Eq. (3.2) the conservation law follows

$$\text{div}\,\mathbf{S} = 0, \tag{3.10}$$

where

$$\mathbf{S}(\mathbf{r}) = \frac{c}{8\pi k_0}\left[\mathbf{k}|E|^2 - \mathbf{E}^*(\mathbf{E}\cdot\mathbf{k})\right] \tag{3.11}$$

is the Poynting vector [22]. For waves (3.9) we get

$$\mathbf{S}_{\pm}^{(j)}(\mathbf{q},z) = \frac{c}{8\pi k_0}A^{(j)2}(\mathbf{q};z,z_0)[\mathbf{k}_{\pm}^{(j)}(\mathbf{q},z) - \mathbf{e}_{\pm}^{(j)}(\mathbf{q},z)(\mathbf{e}_{\pm}^{(j)}(\mathbf{q},z)\cdot\mathbf{k}_{\pm}^{(j)}(\mathbf{q},z))]. \tag{3.12}$$

In our case we have the conservation law in the form $\text{div}\,\mathbf{S} = \partial_z S_z(\mathbf{q},z) = 0$. Therefore the component $S_z(\mathbf{q},z)$ does not depend on z. Then the amplitudes $A^{(j)}(\mathbf{q};z,z_0)$ can be written in the form

$$A^{(j)}(\mathbf{q};z,z_0) = E_0^{(j)}\frac{B^{(j)}(\mathbf{q},z)}{B^{(j)}(\mathbf{q},z_0)}, \tag{3.13}$$

where

$$B^{(j)}(\mathbf{q},z) = \sqrt{\frac{i}{2k_z^{(j)}(\mathbf{q},z)}}\frac{\sqrt{\varepsilon_j}}{n_{(j)}(\mathbf{q},z)\cos\delta_{(j)}(\mathbf{q},z)}, \tag{3.14}$$

$\varepsilon_1 = \varepsilon_\perp$, $\varepsilon_2 = \varepsilon_\parallel$, $\delta_{(j)}(\mathbf{q},z)$ is the angle between the $\mathbf{E}^{(j)}$ and $\mathbf{D}^{(j)} = \hat{\varepsilon}^0\mathbf{E}^{(j)}$ vectors. For the ordinary beam

$$\cos\delta_{(1)} = 1,$$

for the extraordinary beam

$$\cos\delta_{(2)} = \frac{(\mathbf{e}^{(2)}\hat{\varepsilon}^0\mathbf{e}^{(2)})^{1/2}}{n_{(2)}} = \frac{\varepsilon_\perp\sin^2\theta + \varepsilon_\parallel\cos^2\theta}{\sqrt{\varepsilon_\perp^2\sin^2\theta + \varepsilon_\parallel^2\cos^2\theta}}.$$

In this case the constant $E_0^{(j)}$ determines the initial amplitude of the field in the plane $z = z_0$. We omit the signs "$\pm$" if results do not depend on the sign.

Equations (3.6)–(3.14) have a clear physical meaning. They correspond to the adiabatic regime of the wave propagation. These equations can be considered as generalization for the case of the oblique incidence of the well known Mauguin solution [6]. Propagating between planes $z = z_0$ and z the normal wave with index j gains the phase $\int_{z_0}^{z} k_{\pm z}^{(j)}(\mathbf{q},\ z')dz'$. As long as vectors $\mathbf{e}_{\pm}^{(j)}(\mathbf{q},\ z_0)$ and $\mathbf{e}_{\pm}^{(j)}(\mathbf{q},\ z)$ do not coincide the polarization vector rotates in the wave propagation process. The dependence of the amplitudes $A^{(j)}(\mathbf{q};z,z_0)$ on z in Eq. (3.13) is determined by the law of energy conservation for the wave propagating in the inhomogeneous medium without absorption. The wave vectors $\mathbf{k}_{\pm}^{(j)}(\mathbf{q},\ z)$ in each fixed point of CLC are directed normally to the wave front. For the ordinary beam the wave vectors $\mathbf{k}_{\pm}^{(1)} = \mathbf{k}_{\pm}^{(1)}(q)$ do not depend on z, whereas the values and the directions of the wave vectors of the extraordinary beams $\mathbf{k}_{\pm}^{(2)} = \mathbf{k}_{\pm}^{(2)}(\mathbf{q},\ z)$ do depend on z. The directions of polarization vectors $\mathbf{e}_{\pm}^{(j)}(\mathbf{q},\ z)$ depend on z for both types of waves. For each vector $\mathbf{q}$ the wave vectors $\mathbf{k}_{\pm}^{(j)}(\mathbf{q},\ z)$ are in the plane containing vectors $\mathbf{q}$ and $\mathbf{e}_z$ both for ordinary and extraordinary beams.

The tangent to the trajectory of the beam is parallel to the Poynting vector $\mathbf{S}$. Parameterizing the trajectory as $(\mathbf{r}_\perp(z),\ z)$ we can write

$$\frac{d\mathbf{r}_\perp(z)}{dz} = \frac{\mathbf{S}_\perp(z)}{S_z(z)}. \tag{3.15}$$

As far as $\delta_{(1)} = 0$, $\mathbf{S}^{(1)}(\mathbf{q},\ z) \parallel \mathbf{k}^{(1)}(\mathbf{q})$ and it does not depend on z. Therefore the trajectory of the ordinary beam is a straight line parallel to the wave vector $\mathbf{k}^{(1)}$.

In general $\delta_{(2)} \neq 0$ and as it follows from analysis of Eq. (3.12) the vector $\mathbf{S}^{(2)}(\mathbf{q},\ z)$ as a function of z does not belong to the same plane. Since our system is locally uniaxial, $\mathbf{S}^{(2)} \parallel \hat{\varepsilon}^0\mathbf{k}^{(2)}$. So

$$\frac{\mathbf{S}_\perp^{(2)}(z)}{S_z^{(2)}(z)} = \frac{(\hat{\varepsilon}^0(z)\mathbf{k}^{(2)}(z))_\perp}{(\hat{\varepsilon}^0(z)\mathbf{k}^{(2)}(z))_z} \tag{3.16}$$

and Eq. (3.15) for the trajectory of the extraordinary beam designated with sign “$+$” takes the form

$$\frac{d\mathbf{r}_\perp(z)}{dz} = \frac{\mathbf{n}^0(z)q\cos\phi(z)\varepsilon_a + \mathbf{q}\varepsilon_\perp}{k_z^{(2)}(\mathbf{q},\ z)\varepsilon_\perp}. \tag{3.17}$$

Integrating Eq. (3.17) we get the trajectory of the beam

$$\mathbf{r}_\perp(z) = \frac{\varepsilon_a q}{\varepsilon_\perp}\int_0^z \frac{\mathbf{n}^0(z')\cos\phi(z')}{k_z^{(2)}(\mathbf{q},\ z')}dz' + \mathbf{q}\int_0^z \frac{dz'}{k_z^{(2)}(\mathbf{q},\ z')}. \tag{3.18}$$

A typical trajectory of the extraordinary beam calculated by Eq. (3.18) is shown in Figure 2. One can see that the trajectory of the extraordinary beam is helix like.

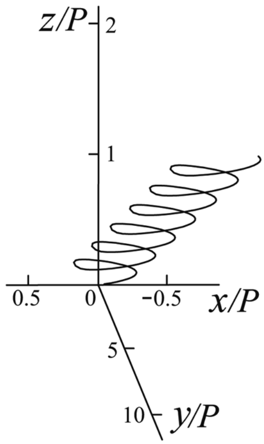


FIGURE 2 A trajectory of the extraordinary beam in CLC. The following parameters were used $\varepsilon_{\parallel} = 2.3$, $\varepsilon_a = 2.0$, the angle of incidence on the plane $z = 0$ is equal to $\pi/4$, and angle $\phi_0 = -\pi/4$. All distances are expressed in terms of P.

Note, if $k_0^2\varepsilon_{\perp} \leq q^2 \leq k_0^2\varepsilon_{\parallel}$ then the extraordinary beam can propagate into the CLC in certain limits of z. The range of these values is determined by the inequality $\cos^2\phi(z) \leq \varepsilon_{\perp}(k_0^2\varepsilon_{\parallel} - q^2)/q^2\varepsilon_a$, for $\varepsilon_a > 0$. If this inequality is violated $k_z^{(2)}$ becomes imaginary and the wave decays exponentially. In this case the capture of the extraordinary beam in CLC takes place [23]. From the physical point of view this effect implies that the beam starts to deviate and in the point $z = z^*(\mathbf{q})$ the component $k_z^{(2)}(\mathbf{q}, z)$ turns to zero changing then its sign. This effect in some aspect is similar to total reflection from a surface inside the medium. Since the refractive index is a periodical function of z such a beam would reflect alternately from two planes normal to the z axis. It means that a plane wave channel is formed and inside this channel the extraordinary beam propagates at a large distance along $\mathbf{r}_{\perp}$ remaining within one period in z. The projection of the ordinary, (1a) and (1b), and extraordinary, (2a), (2b) and (2c), beams and formation of the waveguide propagation are shown in Figure 3.

The trajectory of the ordinary beam is a straight line for an arbitrary angle of incidence. For extraordinary beams with no wave guide regime the trajectory is a helix like (Fig. 2). For extraordinary beams captured into a wave guide channel the trajectory is non-plane also. The effect of the extraordinary beam return was observed experimentally [24]. The setup is analogous to Ref. [4] setup. The LC-cell is shown in Figure 4. The plane parallel layer of LC (1) 100 μm in thickness was

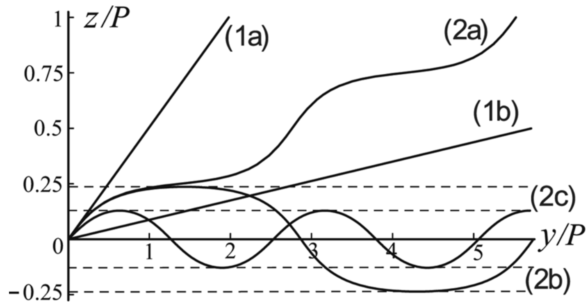


FIGURE 3 The beam trajectories in CLC. Projections on the $z, \mathbf{q}$ plane, $\varepsilon_{\|} = 2.86$, $\varepsilon_{\perp} = 2.28$. All distances are expressed in terms of P.

imbedded between trapezoidal prisms glasses (2 and 3). The incident angle on the LC-glass boundary is denoted as α, the refractive angle inside LC is denoted as χ. The twist structure in the cell appeared to be with the director parallel to the plane of the Figure 4 in the center of the layer and normal to the plane on the orienting surfaces, so the pitch is equal to $P \approx 200\,\mu\text{m}$. We measured the light intensities by detectors Ph_1 and Ph_2 for two polarizations of the incident beam corresponding to the ordinary and extraordinary beams at an entrance of the LC.

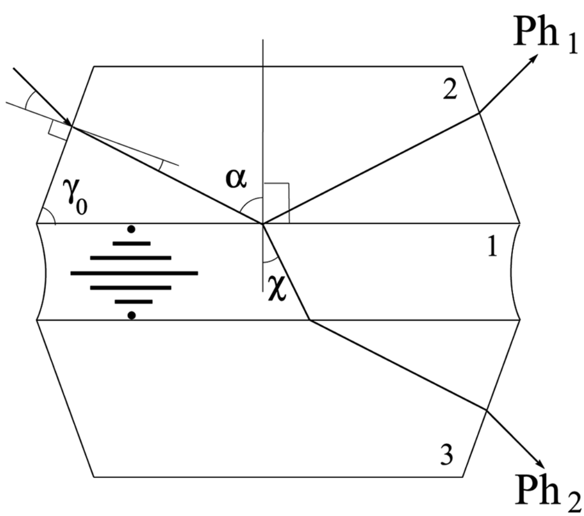


FIGURE 4 The cell with the twisted LC: 1 is LC; 2, 3 are the prism glasses with a height 12 mm, larger base 37 mm and base angle $\gamma_0 = 70°$, LC thickness is $100\,\mu\text{m}$. Ph_1, Ph_2 are photodetectors.

The parameters $\varepsilon_\perp$ and $\varepsilon_\parallel$ are equal to $\varepsilon_\perp = 2.28$ and $\varepsilon_\parallel = 2.86$. Refractive index of the prism is equal to $n_g = 1.644$, i.e., $\sqrt{\varepsilon_\perp} < n_g < \sqrt{\varepsilon_\parallel}$. The angle α varies within the limits $32.54^\circ < \alpha < 90^\circ$.

The obtained experimental and theoretical results are presented in Figures 5 (ordinary beam) and 6 (extraordinary beam). Figures show that for both types of beams there is an angle $\alpha_* \approx 66.6 \div 66.8^\circ$ such that for $\alpha < \alpha_*$ both detectors receive the light and for $\alpha > \alpha_*$ the light is received by the detector Ph_1 only.

As it follows from Snell's law on the boundary glass-LC, $\sqrt{\varepsilon_\perp}\sin\chi = n_g \sin\alpha$, for the ordinary beam there is a restriction on the refractive angle, $\alpha < \arcsin(\sqrt{\varepsilon_\perp}/n_g)$. Substituting n_g and $\sqrt{\varepsilon_\perp}$ values we get $\arcsin(\sqrt{\varepsilon_\perp}/n_g) \approx 66.7^\circ$, i.e., $\alpha_* \approx 66.7^\circ$ is really the angle of the total internal reflection.

For the extraordinary beam Snell's law at the boundary in this case has the form

$$n_g \sin\alpha = n_{(2)}(90^\circ)\sin\chi = \sqrt{\varepsilon_\parallel}\sin\chi. \tag{3.19}$$

As far as $n_g < \sqrt{\varepsilon_\parallel}$ there exists angle χ for any angle α in Eq. (3.19). Therefore there is no total internal reflection on the boundary glass-LC. But the behavior of the extraordinary beam, Figure 6, looks like the behavior of the ordinary beam, Figure 5. Namely for $\alpha < \alpha_* \approx 66.7^\circ$ there are reflected and refracted beams and for $\alpha > \alpha_*$ the refracted beam is absent.

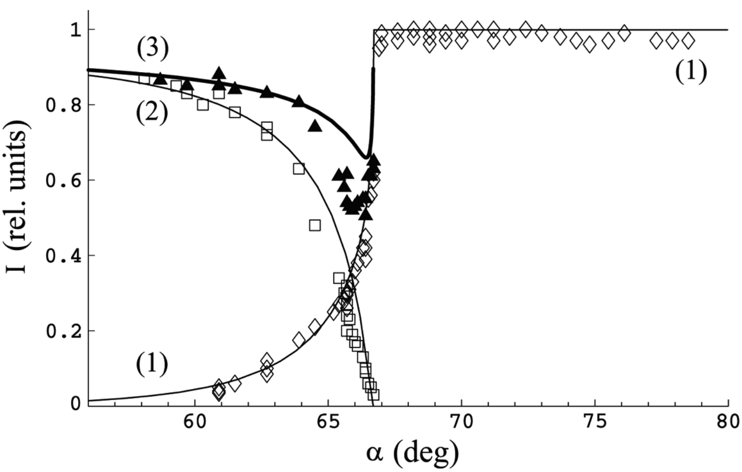


FIGURE 5 The angular dependencies of the intensities of reflected (◇) and refracted (□) ordinary beam; ▲ is the total intensity. Solid lines are calculations: (1) and (2) are $I_r^{(1)}$ and $I_t^{(1)}$, (3) is $I_r^{(1)} + I_t^{(1)}$.

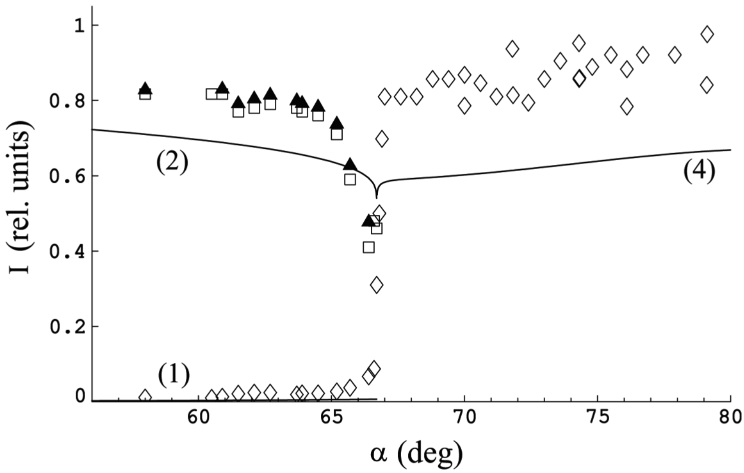


FIGURE 6 The angular dependencies of the intensities of reflected (◇) and refracted (□) extraordinary beam; ▲ is the total intensity. Solid lines are calculations: (1) and (2) are $I_r^{(2)}$ and $I_t^{(2)}$, (4) is $I_r^{(2)}$ for returned beam.

The obtained results can be explained if we take into account the return of the extraordinary beam. According to our calculations for large angles of incidence this beam can not penetrate inside LC more deeply than the layer where $k_z^{(2)} = 0$. Then $k_z^{(2)}$ changes its sign and the beam leaves LC with the same value of the transverse component **q** and opposite value of the longitudinal component $k_z^{(2)}$. Taking into account Snell's law, Eq. (3.19), we get $\sqrt{\varepsilon_\perp}/n_g < \sin\alpha \leq \sqrt{\varepsilon_\parallel}/n_g$. Hence it follows that the return of the extraordinary beam appears in the range of the angles $\alpha_* < \alpha \leq 90^\circ$, as it was observed in the experiment. The return of the beam starts at the angle α wherein $k_z^{(2)}$ becomes equal to zero at the plane with minimal $n_{(2)}$. The minimal $n_{(2)} = \sqrt{\varepsilon_\perp}$ and it coincides with the refraction index $n_{(1)}$ of the ordinary beam.

In theoretical calculations we take into account extinction and neglect the multiple reflections on the boundaries LC-glass. The solid lines in Figure 5 are calculated according to the following expressions

$$I_r^{(1)} = I_0 R_\parallel [1 + a^2(1 - R_\parallel)^2], \quad I_t^{(1)} = I_0 a (1 - R_\parallel)^2, \quad \alpha < \alpha_*, \tag{3.20}$$

$$I_r^{(1)} = I_0, \quad I_t^{(1)} = 0, \quad \alpha > \alpha_*. \tag{3.21}$$

Here I_0 is the intensity of the incident light, $I_r^{(1)}$ and $I_t^{(1)}$ are intensities of reflected and transmitted ordinary beam respectively, $R_\parallel = \tan^2(\chi - \alpha)/\tan^2(\chi + \alpha)$ is the reflection coefficient of the beam with the polarization disposed in the incidence plane [22], $a = \exp[-\int_0^{P/2} \sigma^{(1)}(\theta(z))dl^{(1)}]$, $\sigma^{(1)}(\theta(z))$ is the extinction coefficient of the ordinary beam, $dl^{(1)}$ is the element of the beam trajectory.

Intensities of the reflected, $I_r^{(2)}$, and transmitted, $I_t^{(2)}$, extraordinary beams can be written as

$$\begin{aligned} I_r^{(2)} &= I_0 R_\perp + I_0 R_\perp (1 - R_\perp)^2 \exp(-2\Psi(L)), \\ I_t^{(2)} &= I_0 (1 - R_\perp)^2 \exp(-\Psi(L)), \ \alpha < \alpha_* \end{aligned} \tag{3.22}$$

and

$$I_r^{(2)} = I_0 R_\perp + I_0 (1 - R_\perp)^2 \exp(-2\Psi(z^*)), \quad I_t^{(2)} = 0, \quad \alpha > \alpha_*, \tag{3.23}$$

where $R_\perp = \sin^2(\chi - \alpha)/\sin^2(\chi + \alpha)$ is the reflection coefficient of the beam with the polarization perpendicular to the incidence plane [22], $\Psi(b) = \int_0^b \sigma^{(2)}(\theta(z))dl^{(2)}(z)$, $\sigma^{(2)}(\theta(z))$ is the extinction coefficient of the extraordinary beam, $dl^{(2)}(z)$ is the linear element of the extraordinary beam trajectory. The solid lines in Figure 6 are calculated by Eqs. (3.22), (3.23).

Since our LC is nematic-like for the scale of the order of λ we use the formula for the extinction coefficient in nematic LC [25–27]. The Frank modules are taken as $K_{11} = 1.2 \cdot 10^{-6}$ dyn, $K_{22} = 0.4 \cdot 10^{-6}$ dyn, $K_{33} = 0.99 K_{11}$.

4. THE GREEN'S FUNCTION

The wave equation (3.2) in the integral form is written as

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^0(\mathbf{r}) + k_0^2 \int \widehat{T}^0(\mathbf{r}, \mathbf{r}') \delta\hat{\varepsilon}(\mathbf{r}') \mathbf{E}(\mathbf{r}') d\mathbf{r}'. \tag{4.1}$$

The electric field $\mathbf{E}^0(\mathbf{r})$ is the solution of the wave equation (3.2) for equilibrium permittivity $\hat{\varepsilon}^0$. The Green's function of electromagnetic field $\widehat{T}^0(\mathbf{r}, \ \mathbf{r}')$ obeys the equations

$$(\text{curl curl} - k_0^2 \hat{\varepsilon}^0(z)) \widehat{T}^0(\mathbf{r}, \ \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}') \widehat{I}. \tag{4.2}$$

Here $\widehat{I}$ is the unit matrix.

As far as our medium is homogeneous in the *xy* plane it is suitable to perform the transverse Fourier transformation. The corresponding problem (4.2) for the field of the point source $\widehat{T}^0(\mathbf{q}; z, \ z_1)$ is reduced to a set of equations

$$\widehat{\mathcal{L}}(z) \widehat{T}^0(\mathbf{q}; z, \ z_1) = \delta(z - z_1) \widehat{I}, \tag{4.3}$$

where

$$\mathcal{L}_{\alpha\beta}(z) = (e_{z\alpha}e_{z\beta} - \delta_{\alpha\beta})\frac{\partial^2}{\partial z^2} + i(q_\alpha e_{z\beta} + q_\beta e_{z\alpha})\frac{\partial}{\partial z} + (q^2\delta_{\alpha\beta} - q_\alpha q_\beta) - k_0^2\varepsilon^0_{\alpha\beta}(z)$$

is the linear differential operator of the second order. For convenience we consider here $\mathbf{q}$ as a three-dimensional vector with q_z component being equal to zero, $q_z = 0$. Thus the calculation of the Green's function leads to solution of the system of nine differential equations with periodic coefficients. The principle of radiation determining the behavior of the solution at $z \to +\infty$ and $z \to -\infty$ is chosen as boundary conditions. The solution of the inhomogeneous equation (4.3) is constructed as the superposition of solutions of the corresponding homogeneous equation (3.9) in the regions $z > z_1$ and $z < z_1$. The coefficients of the superposition are chosen so as to ensure the corresponding singularity in the right hand side of Eq. (4.3). The asymptotics of the Green's function in the far zone for the case $\Omega \gg 1$ has the form [23]

$$\widehat{T}^0(\mathbf{q}; z, z_1) = \widehat{T}^{(1)}(\mathbf{q}; z, z_1) + \widehat{T}^{(2)}(\mathbf{q}; z, z_1), \tag{4.4}$$

where

$$T^{(j)}_{\alpha\beta}(\mathbf{q}; z, z_1) = B^{(j)}(\mathbf{q}, z)B^{(j)}(\mathbf{q}, z_1)e^{(j)}_\alpha(\mathbf{q}, z)e^{(j)}_\beta(\mathbf{q}, z_1) \times \exp\left(i\left|\int_{z_1}^{z} k_z^{(j)}(\mathbf{q}, z')dz'\right|\right), \tag{4.5}$$

for $j = 1, 2$. Polarizations in Eq. (4.5) are chosen in the following way. For $z \geq z_1$ we have

$$e^{(j)}_\alpha(\mathbf{q}, z) = e^{(j)}_{+\alpha}(\mathbf{q}, z), \quad e^{(j)}_\beta(\mathbf{q}, z_1) = e^{(j)}_{-\beta}(\mathbf{q}, z_1).$$

For $z < z_1$, we have

$$e^{(j)}_\alpha(\mathbf{q}, z) = e^{(j)}_{-\alpha}(\mathbf{q}, z), \quad e^{(j)}_\beta(\mathbf{q}, z_1) = e^{(j)}_{+\beta}(\mathbf{q}, z_1).$$

In order to get the Green's function in the coordinate presentation it is necessary to perform inverse two-dimensional Fourier transformation. The integrals can be calculated by the stationary phase method.

Let us estimate the fraction of the energy C_t of the extraordinary wave captured in the channel from the point-like source situated in the origin of the coordinate frame (see Fig. 3 beams (2b) and (2c)). This value is of the order of the fraction of the solid angle θ_t forming by the beams outgoing into the channel to the total solid angle 4π. For positive directions, $z > 0$ the beams in the channel are radiated in the range of angles $\chi_0^* \leq \chi \leq \pi/2$ where the minimal angle χ_0^* is determined by condition $n_{(2)} = \sqrt{\varepsilon_\perp}$. The solid angle θ_t has the form

$$\theta_t = 2\int_0^{2\pi} d\phi_i \left(\int_{\chi_0^*}^{\pi/2} \sin\chi \, d\chi\right). \tag{4.6}$$

Here factor 2 is introduced in order to take into account beams propagating in the negative direction, $z < 0$. Calculating integrals (4.6) we get

$$C_t \sim \frac{\theta_t}{4\pi} = \frac{2}{\pi}\arctan\frac{\sqrt{\varepsilon_a}}{\sqrt{\varepsilon_\perp}}. \tag{4.7}$$

For $\varepsilon_\perp = 2.28$ and $\varepsilon_\| = 2.86$ used in calculation, Figure 3, the fraction of the energy outgoing into the wave guide channel is $C_t \approx 0.30$.

5. SINGLE LIGHT SCATTERING

The second term in the right hand side of Eq. (4.1) corresponds to the scattered field $\mathbf{E}^{(s)}$, produced by the incident field $\mathbf{E}^0$. Solving this equation by iterations and restricting ourselves to the lowest order in $\delta\hat{\varepsilon}$ we obtain the scattered field $\mathbf{E}^{(s)}$ in the Born (single-scattering) approximation

$$\mathbf{E}^{(s)}(\mathbf{r}) = k_0^2 \int \widehat{T}^0(\mathbf{r}_\perp - \mathbf{r}'_\perp; z,\ z')\delta\hat{\varepsilon}(\mathbf{r}')\mathbf{E}^0(\mathbf{r}')d\mathbf{r}'. \tag{5.1}$$

In the first order the permittivity fluctuations in CLC have the form

$$\delta\varepsilon_{\alpha\beta}(\mathbf{r}) = \varepsilon_a(n_\alpha^0(z)\delta n_\beta(\mathbf{r}) + \delta n_\alpha(\mathbf{r})n_\beta^0(z)). \tag{5.2}$$

The properties of the scattered light are determined by the function of coherence

$$\langle E_\alpha^{(s)}(\mathbf{r}_1)E_\beta^{(s)*}(\mathbf{r}_2)\rangle = k_0^4 \int T_{\alpha\gamma}^0(\mathbf{r}_{1\perp} - \mathbf{r}'_{1\perp}; z_1,\ z'_1)T_{\beta\zeta}^{0*}(\mathbf{r}_{2\perp} - \mathbf{r}'_{2\perp}; z_2,\ z'_2)$$
$$\times\, \mathcal{G}_{\gamma\nu\zeta\mu}(\mathbf{r}'_1,\ \mathbf{r}'_2)\, E_\nu^0(\mathbf{r}'_1)E_\mu^{0*}(\mathbf{r}'_2)d\mathbf{r}'_1 d\mathbf{r}'_2, \tag{5.3}$$

where

$$\mathcal{G}_{\gamma\nu\zeta\mu}(\mathbf{r}'_1,\ \mathbf{r}'_2) = \langle\delta\varepsilon_{\gamma\nu}(\mathbf{r}'_1)\delta\varepsilon_{\zeta\mu}^*(\mathbf{r}'_2)\rangle$$

is the permittivity correlation function. Due to CLC symmetry we have

$$\widehat{\mathcal{G}}(\mathbf{r}'_1, \mathbf{r}'_2) \equiv \widehat{\mathcal{G}}(\mathbf{r}'_{1\perp} - \mathbf{r}'_{2\perp}; z'_1,\ z'_2).$$

Thus for calculation of the coherence function (5.3) of the system it is necessary to know the normal waves which determine the field $\mathbf{E}^0$, the Green's function $\widehat{T}^0$, and the correlation function of the permittivity fluctuations $\widehat{\mathcal{G}}$.

The relationship between correlation function of the permittivity fluctuations and correlation function of the director fluctuations (2.3) is:

$$\begin{aligned}\mathcal{G}_{\alpha\beta\gamma\delta}(\mathbf{r}_\perp;\ z,\ z') = \varepsilon_a^2[&n^0_\alpha(z)n^0_\gamma(z')g_{\beta\delta}(\mathbf{r}_\perp; z,\ z') + n^0_\alpha(z)n^0_\delta(z')g_{\beta\gamma}(\mathbf{r}_\perp; z,\ z')\\ &+ n^0_\beta(z)n^0_\gamma(z')g_{\alpha\delta}(\mathbf{r}_\perp; z,\ z') + n^0_\beta(z)n^0_\delta(z')g_{\alpha\gamma}(\mathbf{r}_\perp; z,\ z')].\end{aligned} \tag{5.4}$$

For the scattering medium with the periodic inhomogeneities and the homogeneous environment the normal waves and the Green's function inside and outside the scattering volume are essentially different. In particular the incident and the scattered waves can be considered as plane waves outside the medium only, so the effect of the boundary is important.

In order to overcome this obstacle for the scattering problem in the medium with one-dimensional inhomogeneities we suggest the Kirchhoff method. The problem is solved in three stages. 1. The scattered field is calculated inside the medium in the ($\mathbf{q}$, z) representation. 2. We recalculate the scattered field in the boundary outside the medium into that in the outside space. 3. The coordinate representation of the field in the outside area is calculated on the basis of its value in the boundary outside the medium in ($\mathbf{q}$, z) representation.

We assume that the scattering volume is the plane layer, $0 \le z \le L$, with a large transverse size $L_\perp \gg L$ (Fig. 7). The incident plane wave starts from $z = -\infty$ and the scattered field is recorded in the region $z > L$, i.e., in the positive half-space. The latter is not essential as far as scattering in the negative half-space, $z < 0$, can be considered in a similar way. Let the incident field be a plane wave with the wave vector $\mathbf{k}^{(i)}$. The scattered wave has the wave vector $\mathbf{k}^{(s)}$ and is measured in the far zone. The scattering volume is embedded in homogeneous environment with permittivity ε_0.

Due to the identity $k_z^2 + k_\perp^2 = k_0^2\varepsilon_0$ which is valid outside the stratified medium the total wave vector $\mathbf{k}$ is determined by the component $\mathbf{k}_\perp$ and the sign of the k_z component. Therefore it is sufficient to define the vector $\mathbf{k}_\perp^{(i)}$ and direction of the incident wave, positive or negative, with respect to z. The latter is valid for the wave vector $\mathbf{k}_\perp^{(s)}$ too.

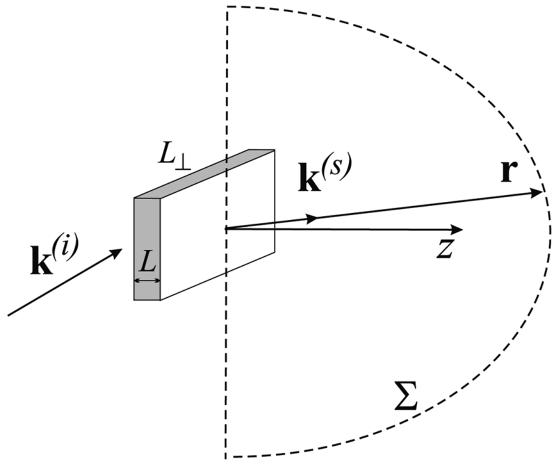


FIGURE 7 Geometry of light scattering.

Below indices "*in*" and "*out*" refer to the values inside and outside of the inhomogeneous medium respectively.

5.1. Kirchhoff Method

Let us consider an arbitrary inhomogeneous specimen Γ bordering with closed surface Σ situated outside in a homogeneous medium. The electromagnetic field outside the inhomogeneous specimen, $\mathbf{E}(\mathbf{r}) = \mathbf{E}_{out}(\mathbf{r})$, satisfies the wave equation

$$(\text{curl curl} - k_0^2\varepsilon_0)\mathbf{E}(\mathbf{r}) = 0. \tag{5.5}$$

It is easy to notice that the system of the three equations (5.5) is equivalent to the system

$$\begin{cases} (\Delta + k_0^2\varepsilon_0)E_\alpha(\mathbf{r}) = 0 \\ \text{div}\mathbf{E}(\mathbf{r}) = 0. \end{cases} \tag{5.6}$$

The second equation means that the electromagnetic field is transversal.

The Green's function $T(\mathbf{r},\ \mathbf{r}') = T_{out}(\mathbf{r},\ \mathbf{r}'),\ \mathbf{r}, \mathbf{r}' \notin \Gamma$, for each component $E_\alpha(\mathbf{r})$ satisfies the equation

$$(\Delta + k_0^2\varepsilon_0)T(\mathbf{r},\ \mathbf{r}') = -\delta(\mathbf{r} - \mathbf{r}'). \tag{5.7}$$

Equation (5.7) does not define the function $T(\mathbf{r},\ \mathbf{r}')$ uniquely and additional boundary conditions are required. If we take $T|_\Sigma = 0$ as the boundary condition then the field $E_\alpha(\mathbf{r})$ in the observation point may be expressed in terms of the field $E_\alpha(\mathbf{r}')$ on the surface Σ. According to the Kirchhoff-Helmholtz integral theorem [28],

$$E_\alpha(\mathbf{r}) = -\int_\Sigma d^2r' E_\alpha(\mathbf{r}')\nabla_{\mathbf{r}'} T(\mathbf{r},\ \mathbf{r}')\cdot \mathbf{s}(\mathbf{r}'). \tag{5.8}$$

where $\mathbf{s}(\mathbf{r}')$ is the external normal to the surface Σ in the $\mathbf{r}'$ point.

The expression for the Green's function satisfying the condition $T|_\Sigma = 0$ depends on the form of the specimen. For simplicity we shall consider the surface Σ as a piece of the plane $z = L$ with a large transverse size $L_\perp$ closed by a large hemisphere. If the Green's function $T(\mathbf{r},\ \mathbf{r}')$ satisfies the radiation condition in the infinity then the contribution of the hemisphere to integral (5.8) tends to zero with increasing of its radius. In this case the boundary condition $T|_\Sigma = 0$ in our geometry is reduced to $T|_{z=L} = 0$, and using the mirror image method we get

$$T(\mathbf{r},\ \mathbf{r}') = \frac{1}{4\pi}\left(\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} - \frac{e^{ik|\mathbf{r}-\mathbf{r}'_1|}}{|\mathbf{r}-\mathbf{r}'_1|}\right), \tag{5.9}$$

where $\mathbf{r}'_1$ is the mirror image of $\mathbf{r}'$ point with respect to the boundary plane $z = L$.

Let us suppose that the field is measured in the point $\mathbf{r} = (\mathbf{r}_\perp,\ z),\ z - L \gg L_\perp$. Then in both terms of Eq. (5.9) we can use the plane wave approximation of the form

$$\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \approx \frac{e^{ikr}}{r} e^{-i\mathbf{k}^{(s)}\cdot\mathbf{r}'}. \tag{5.10}$$

As far as in our geometry $\mathbf{s}(\mathbf{r}')\cdot\nabla_{\mathbf{r}'} = -\partial/\partial z'$, we get from Eq. (5.8)

$$\mathbf{E}(\mathbf{r}) = \frac{-ik_0\sqrt{\varepsilon_0}}{2\pi}\frac{e^{ikr}}{r}\frac{z}{r} e^{-ik_z^{(s)}L}\widehat{P}(\mathbf{r})\mathbf{E}(\mathbf{k}_\perp^{(s)},\ L). \tag{5.11}$$

In order to fulfil the condition div $\mathbf{E} = 0$ we multiplied the field by the projector

$$\widehat{P}(\mathbf{r}) = \widehat{I} - \frac{\mathbf{r}\otimes\mathbf{r}}{r^2},$$

providing the field to be transversal in the far zone. This way we get the vector analog of the Kirchhoff formula.

Now due to relation $e_\alpha^{(s)}P_{\alpha\beta} = e_\beta^{(s)}$ we obtain the intensity of the scattered field with polarization $\mathbf{e}^{(s)}$

$$I = \frac{\sqrt{\varepsilon_0}c}{8\pi}\frac{k_0^2\varepsilon_0}{4\pi^2}\frac{1}{r^2}\left(\frac{z}{r}\right)^2\left\langle\left|\mathbf{e}^{(s)}\cdot\mathbf{E}_{out}^{(s)}(\mathbf{k}_\perp^{(s)}, L)\right|^2\right\rangle. \tag{5.12}$$

Thus the scattered field in ($\mathbf{q}$, z) representation in the boundary outside the media, $\mathbf{E}^{(s)}_{out}$, is required for the following calculations.

5.2. Boundary Conditions

The transverse components of the wave vectors and the fields do not change when the waves pass through the boundaries

$$\begin{aligned}\mathbf{E}^{(i)}_{out\perp}(\mathbf{k}^{(i)}_{\perp},\ 0) &= \mathbf{E}^{(i)}_{in\perp}(\mathbf{k}^{(i)}_{\perp},\ 0),\\ \mathbf{E}^{(s)}_{out\perp}(\mathbf{k}^{(s)}_{\perp},\ L) &= \mathbf{E}^{(s)}_{in\perp}(\mathbf{k}^{(s)}_{\perp},\ L).\end{aligned} \tag{5.13}$$

The z components of the fields could be obtained from the condition for the induction vector div$\mathbf{D} = 0$. This condition gives

$$\begin{aligned}D^{(i)}_{out\,z}(\mathbf{k}^{(i)}_{\perp},\ 0) &= D^{(i)}_{in\,z}(\mathbf{k}^{(i)}_{\perp},\ 0),\\ D^{(s)}_{out\,z}(\mathbf{k}^{(s)}_{\perp},\ L) &= D^{(s)}_{in\,z}(\mathbf{k}^{(s)}_{\perp},\ L).\end{aligned} \tag{5.14}$$

So we can write the boundary conditions in the form

$$\begin{aligned}\mathbf{E}^{(i)}_{in}(\mathbf{k}^{(i)}_{\perp},\ 0) &= \widehat{M}^{out\to in}(\mathbf{k}^{(i)}_{\perp},\ 0)\mathbf{E}^{(i)}_{out}(\mathbf{k}^{(i)}_{\perp},\ 0),\\ \mathbf{E}^{(s)}_{out}(\mathbf{k}^{(s)}_{\perp},\ L) &= \widehat{M}^{in\to out}(\mathbf{k}^{(s)}_{\perp},\ L)\mathbf{E}^{(s)}_{in}(\mathbf{k}^{(s)}_{\perp},\ L).\end{aligned} \tag{5.15}$$

Here $\widehat{M}^{out\to in}$ and $\widehat{M}^{in\to out}$ are transformation matrices which can be calculated from Eqs. (5.13) and (5.14). We get

$$\begin{aligned}\widehat{M}^{out\to in} &= \widehat{\mathrm{diag}}(1,\ 1,\ \varepsilon_0/\varepsilon_{\perp}),\\ \widehat{M}^{in\to out} &= (\widehat{M}^{out\to in})^{-1},\end{aligned} \tag{5.16}$$

where $\widehat{\mathrm{diag}}$ denotes the diagonal matrix.

5.3. Scattering of Light

For stratified media with boundaries parallel to the layers the wave inside the medium has the form

$$\mathbf{E}^{(i)}_{in}(\mathbf{r}) = \boldsymbol{\mathcal{E}}^{(i)}(\mathbf{k}^{(i)}_{\perp},\ z)e^{i\mathbf{k}^{(i)}_{\perp}\cdot\mathbf{r}_{\perp}}, \tag{5.17}$$

where $\boldsymbol{\mathcal{E}}^{(i)}(\mathbf{k}^{(i)}_{\perp},\ z)$ is determined by the properties of the stratified medium and also by polarization and amplitude of the incident wave. In case $P \gg \lambda$ the function $\boldsymbol{\mathcal{E}}^{(i)}$ is determined by Eqs. (3.9), (3.13).

From Eqs. (5.1), (5.17) we obtain the scattered field $\mathbf{E}^{(s)}_{in}(\mathbf{k}^{(s)}_{\perp},\ L)$ at the boundary $z = L$ inside the specimen

$$\mathbf{E}_{in}^{(s)}(\mathbf{k}_\perp^{(s)},\ L) = k_0^2 \int_0^L dz' \widehat{T}^0(\mathbf{k}_\perp^{(s)}; L, z')\delta\hat{\varepsilon}(\mathbf{k}_\perp^{(s)} - \mathbf{k}_\perp^{(i)},\ z')\boldsymbol{\mathcal{E}}^{(i)}(\mathbf{k}_\perp^{(i)},\ z'). \tag{5.18}$$

From Eqs. (5.15), (5.18) and relation $\langle \delta\hat{\varepsilon}(\mathbf{k}_\perp,\ z) \otimes \delta\hat{\varepsilon}^*(\mathbf{k}_\perp,\ z')\rangle = S_\perp \widehat{\mathcal{G}}(\mathbf{k}_\perp; z,\ z')$ we obtain the intensity of the single light scattering outside the specimen (5.12) in the form

$$\begin{aligned} I = {} & \frac{\sqrt{\varepsilon_0}c}{8\pi}\frac{k_0^6\varepsilon_0}{4\pi^2}\frac{S_\perp}{r^2}\left(\frac{z}{r}\right)^2 e_\alpha^{(s)} e_\gamma^{(s)} M_{\alpha\beta}^{in\to out} M_{\gamma\delta}^{in\to out} \int_0^L dz_1 \int_0^L dz_2 \\ & \times T_{\beta\rho}^0(\mathbf{k}_\perp^{(s)}; L,\ z_1)\ T_{\delta\varphi}^{0*}(\mathbf{k}_\perp^{(s)}; L,\ z_2) \mathcal{G}_{\rho\nu\varphi\mu}(\mathbf{k}_\perp^{(s)} - \mathbf{k}_\perp^{(i)}; z_1,\ z_2) \\ & \times \mathcal{E}_\nu^{(i)}(\mathbf{k}_\perp^{(i)},\ z_1)\mathcal{E}_\mu^{(i)*}(\mathbf{k}_\perp^{(i)},\ z_2), \end{aligned} \tag{5.19}$$

where $S_\perp$ is the cross-section area of the specimen.

We restrict our treatment to the case when the polarization of the incident light inside the medium has only one of two possible types of waves $\mathbf{E}_{in}^{(i)}(\mathbf{r})$, Eq. (3.9). Otherwise the summation over polarizations (i) should be performed for the field inside the medium. In a similar way the scattered light outside the medium corresponds only to one type of the scattered wave inside the medium, $\mathbf{E}_{in}^{(s)}(\mathbf{r})$. So in what follows it is possible to omit the summation over (s) for the field inside the medium. Thus indices (i) and (s) take values 1, 2 dependent on types of the incident and scattered waves.

6. LIGHT SCATTERING INTENSITY IN CLC WITH LARGE PITCH

Substituting expressions for the incident field, Eqs. (3.9), and the Green's function, Eq. (4.5), into Eq. (5.19) we get the scattering intensity

$$\begin{aligned} I_{(i)}^{(s)} = {} & J_0\left(\frac{z}{r}\right)^2 |B^{(s)}(\mathbf{k}_\perp^{(s)},\ L)|^2 \frac{V_{sc}}{r^2 L}\int_0^L dz_1 \int_0^L dz_2 \exp\left[i\int_{z_1}^{z_2} q_z^{(sc)}(z')dz'\right] \\ & \times A^{(i)}(\mathbf{k}_\perp^{(i)},\ z_1)A^{(i)*}(\mathbf{k}_\perp^{(i)},\ z_2)B^{(s)}(\mathbf{k}_\perp^{(s)},\ z_1)B^{(s)*}(\mathbf{k}_\perp^{(s)},\ z_2) \\ & \times e_\rho^{(s)}(\mathbf{k}_\perp^{(s)},\ z_1)e_\varphi^{(s)}(\mathbf{k}_\perp^{(s)},\ z_2)\mathcal{G}_{\rho\nu\varphi\mu}(\mathbf{q}_\perp^{(sc)}; z_1,\ z_2)e_\nu^{(i)}(\mathbf{k}_\perp^{(i)},\ z_1) \\ & \times e_\mu^{(i)}(\mathbf{k}_\perp^{(i)},\ z_2), \end{aligned} \tag{6.1}$$

where $\mathbf{q}_\perp^{(sc)} = \mathbf{k}_\perp^{(s)} - \mathbf{k}_\perp^{(i)}$, $q_z^{(sc)}(z) = k_z^{(s)}(\mathbf{k}_\perp^{(s)},\ z) - k_z^{(i)}(\mathbf{k}_\perp^{(i)},\ z)$,

$$J_0 = \frac{\sqrt{\varepsilon_0}c}{8\pi}\frac{k_0^6\varepsilon_0}{4\pi^2}e_\alpha^{(s)}e_\gamma^{(s)}M_{\alpha\beta}^{in\to out}(\mathbf{k}_\perp^{(s)},\ L)M_{\gamma\delta}^{in\to out}(\mathbf{k}_\perp^{(s)},\ L)$$
$$\times e_\beta^{(s)}(\mathbf{k}_\perp^{(s)},\ L)e_\delta^{(s)}(\mathbf{k}_\perp^{(s)},\ L)$$

and $V_{sc} = S_\perp L$ is the scattering volume. Using the first equation in (5.16) we can calculate the field $E_{0\,in}^{(i)}$ inside the medium through the field $E_{0\,out}^{(i)}$ outside the medium:

$$E_{0\,in}^{(i)} = E_{0\,out}^{(i)}\left[e_{out\perp}^{(i)2} + \frac{\varepsilon_0^2}{\varepsilon_\perp^2}e_{out\,z}^{(i)2}\right]^{-1/2},$$

where $\mathbf{e}_{out}^{(i)}$ is the polarization vector of the incident field outside the medium.

Integral (6.1) contains rapidly oscillating factor $\exp\left[i\int_{z_1}^{z_2}q_z^{(sc)}(z')dz'\right]$. As far as the vicinity of the line $z_1 = z_2$ yields the main contribution to the asymptotic behavior of the integral (6.1) it is convenient to introduce new variables $z_+ = (z_1 + z_2)/2$ and $z_- = z_2 - z_1$. Expanding the phase function in series near the line $z_- = 0$ up to the terms of the first order we have

$$\int_{z_+-z_-/2}^{z_++z_-/2} q_z^{(sc)}(z')dz' \approx q_z^{(sc)}(z_+)z_-. \tag{6.2}$$

This approach is valid for $q_z^{(sc)}P \gg 1$. The correlation function (2.19) contains rapidly decaying factors $\exp\left[-q_\perp^{(sc)}\left|\int_{z_1}^{z_2}\mu_j(z)dz\right|\right]$. Therefore the approach is valid not only for $q_z^{(sc)}P \gg 1$ but also for $q_z^{(sc)}P \sim 1$, $q_\perp^{(sc)}P \gg 1$, i.e., finally for $q^{(sc)}P \gg 1$.

Functions $A^{(i)}(\mathbf{k}_\perp^{(i)},\ z)$, $B^{(s)}(\mathbf{k}_\perp^{(s)},\ z)$ and $e_\beta^{(s,i)}(\mathbf{k}_\perp^{(s,i)},\ z)$ vary slowly compared to the rapidly oscillating function $\exp[iq_z^{(sc)}(z_+)z_-]$. Therefore it is possible to substitute z_+ instead of z_1 and z_2 into these functions. We can expand the region of integration over the z_- variable within the limits $\pm\infty$ and for the correlation function we get the Fourier image $\hat{\mathcal{G}}(\mathbf{q}_\perp^{(sc)},\ q_z^{(sc)}(z_+),\ z_+)$.

Thus the intensity of light scattering has the form

$$I_{(i)}^{(s)} = J_0\left(\frac{z}{r}\right)^2|B^{(s)}(\mathbf{k}_\perp^{(s)},\ L)|^2\frac{V_{sc}}{r^2L}\int_0^L dz_+|A^{(i)}(\mathbf{k}_\perp^{(i)},\ z_+)|^2|B^{(s)}(\mathbf{k}_\perp^{(s)},\ z_+)|^2$$
$$\times e_\rho^{(s)}(\mathbf{k}_\perp^{(s)},\ z_+)e_\varphi^{(s)}(\mathbf{k}_\perp^{(s)},\ z_+)\mathcal{G}_{\rho\nu\varphi\mu}(\mathbf{q}^{(sc)}(z_+),\ z_+)$$
$$\times e_\nu^{(i)}(\mathbf{k}_\perp^{(i)},\ z_+)e_\mu^{(i)}(\mathbf{k}_\perp^{(i)},\ z_+), \tag{6.3}$$

where $\mathbf{q}^{(sc)}(z) = (\mathbf{q}_\perp^{(sc)},\ q_z^{(sc)}(z))$.

In all slowly varying factors of $\hat{\mathcal{G}}(\mathbf{q}; z_1,\ z_2)$ we substitute z_1 and z_2 by z_+. We restrict ourselves by the term linear in z_- in the exponential factors. In this case Eq. (5.4) takes the form

$$\begin{aligned}\mathcal{G}_{\alpha\beta\gamma\delta}(\mathbf{q}^{(sc)}(z),\ z) = \varepsilon_a^2[&n_\alpha^0(z)n_\gamma^0(z)g_{\beta\delta}(\mathbf{q}^{(sc)}(z),\ z)\\ &+n_\alpha^0(z)n_\delta^0(z)g_{\beta\gamma}(\mathbf{q}^{(sc)}(z),\ z)\\ &+n_\beta^0(z)n_\gamma^0(z)g_{\alpha\delta}(\mathbf{q}^{(sc)}(z),\ z)\\ &+n_\beta^0(z)n_\delta^0(z)g_{\alpha\gamma}(\mathbf{q}^{(sc)}(z),\ z)],\end{aligned} \tag{6.4}$$

where

$$\begin{aligned}g_{\alpha\beta}(\mathbf{q}^{(sc)}(z),\ z) = &\frac{k_BT}{2q_\perp^{(sc)}K_{33}\cos^2\phi(z)}\sum_{j=1}^{2}\int_{-\infty}^{\infty}dz_-\\ &\times\exp\left(iq_z^{(sc)}(z)z_- - q_\perp^{(sc)}\mu_j(z)|z_-|\right)\\ &\times f_\alpha^{(j)}(\mathbf{q}_\perp^{(sc)}; z,\ z_-)f_\beta^{(j)}(\mathbf{q}_\perp^{(sc)}; z,\ z_-).\end{aligned} \tag{6.5}$$

Performing integration over z_- and summation over j in Eq. (6.5) we get

$$g_{\alpha\beta}(\mathbf{q}^{(sc)}(z),\ z) = k_BT\sum_{j=1}^{2}\frac{e_{j\alpha}(\mathbf{q}^{(sc)},\ z)e_{j\beta}(\mathbf{q}^{(sc)},\ z)}{K_{jj}[q^{(sc)2} - (\mathbf{q}^{(sc)}\cdot\mathbf{n}^0)^2] + K_{33}(\mathbf{q}^{(sc)}\cdot\mathbf{n}^0)^2}, \tag{6.6}$$

where

$$\begin{aligned}\mathbf{e}_2(\mathbf{q}^{(sc)}(z),\ z) &= \frac{\mathbf{q}^{(sc)}\times\mathbf{n}^0}{\sqrt{q^{(sc)2} - (\mathbf{q}^{(sc)}\cdot\mathbf{n}^0)^2}},\\ \mathbf{e}_1(\mathbf{q}^{(sc)}(z),\ z) &= \mathbf{n}^0\times\mathbf{e}_2(\mathbf{q}^{(sc)},\ z).\end{aligned} \tag{6.7}$$

Substituting Eq. (6.6) into Eq. (6.4) we get the correlation function of permittivity fluctuation in CLC

$$\begin{aligned}\mathcal{G}_{\rho\nu\varphi\mu}(\mathbf{q}^{(sc)}(z), z) = &\sum_{j=1}^{2}\frac{k_BT\varepsilon_a^2}{K_{jj}q^{(sc)2} + (K_{33} - K_{jj})(\mathbf{q}^{(sc)}\cdot\mathbf{n}^0(z))^2}\\ &\times(e_{j\nu}(\mathbf{q}^{(sc)},\ z)\mathbf{n}_\rho^0(z)\\ &+e_{j\rho}(\mathbf{q}^{(sc)},\ z)\mathbf{n}_\nu^0(z))(e_{j\mu}(\mathbf{q}^{(sc)},\ z)\mathbf{n}_\varphi^0(z)\\ &+e_{j\varphi}(\mathbf{q}^{(sc)},\ z)\mathbf{n}_\mu^0(z)).\end{aligned} \tag{6.8}$$

Thus in the problem of light scattering it is possible to restrict ourself to the expression (6.8) for the correlation function if the inequalities

$$q \gg q_0, \quad |z_1 - z_2| \ll q/q_0^2 \tag{6.9}$$

are fulfilled.

Substituting Eq. (6.8) into Eq. (6.3) we get for the light scattering intensity

$$\begin{aligned} I_{(i)}^{(s)} \equiv I(\mathbf{e}^{(i)}, \mathbf{e}^{(s)}) = {} & J_0 k_B T \varepsilon_a^2 \left(\frac{z}{r}\right)^2 |B^{(s)}(\mathbf{k}_\perp^{(s)}, L)|^2 \frac{V_{sc}}{r^2 L} \int_0^L dz \\ & \times \sum_{j=1,2} \frac{|A^{(i)}(\mathbf{k}_\perp^{(i)}, z)|^2 |B^{(s)}(\mathbf{k}_\perp^{(s)}, z)|^2}{K_{jj} q^{(sc)2} + (K_{33} - K_{jj})(\mathbf{q}^{(sc)} \cdot \mathbf{n}^0)^2} \\ & \times \Big[(\mathbf{e}_j \cdot \mathbf{e}^{(i)})(\mathbf{n}^0 \cdot \mathbf{e}^{(s)}) \\ & + (\mathbf{e}_j \cdot \mathbf{e}^{(s)})(\mathbf{n}^0 \cdot \mathbf{e}^{(i)})\Big]^2, \end{aligned} \tag{6.10}$$

where $\mathbf{n}^0 = \mathbf{n}^0(z), \mathbf{e}^{(i)} = \mathbf{e}^{(i)}(\mathbf{k}_\perp^{(i)},\ z), \mathbf{e}^{(s)} = \mathbf{e}^{(s)}(\mathbf{k}_\perp^{(s)},\ z), \mathbf{e}_j = \mathbf{e}_j(\mathbf{q}^{(sc)}, z)$, $\mathbf{q}^{(sc)} = \mathbf{q}^{(sc)}(z)$. Comparing the applicability conditions for Eqs. (6.3) and (6.8) we finally get that Eq. (6.10) is valid for $q_\perp^{(sc)} P \gg 1$.

6.1. The Basic Scattering Geometries

Let us analyze the light scattering intensities for various polarizations. In what follows we use the notations (o) and (e) for ordinary and extraordinary beams. In this system there exist four types of scattering, (i)–(s).

The scattering of the (o)–(o) type is absent since the polarization vector of the ordinary beam is perpendicular to the director, $\mathbf{n}^0 \cdot \mathbf{e}^{(1)} = 0$. So for the (o)–(o) scattering it is valid that $\mathbf{n}^0 \cdot \mathbf{e}^{(i)} = 0,\ \mathbf{n}^0 \cdot \mathbf{e}^{(s)} = 0$ and hence the scattering intensity (6.10) goes to zero. This situation is similar to that for the nematic liquid crystal.

In the case of (o)–(e) scattering there is only one nonzero term in brackets of Eq. (6.10). So we get

$$\begin{aligned} I(\mathbf{e}^{(1)}, \mathbf{e}^{(2)}) = {} & J_0 k_B T \varepsilon_a^2 \left(\frac{z}{r}\right)^2 |B^{(2)}(\mathbf{k}_\perp^{(s)}, L)|^2 E_0^{(1)2} \frac{V_{sc}}{r^2 L} \int_0^L dz \\ & \times |B^{(2)}(\mathbf{k}_\perp^{(s)},\ z)|^2 (\mathbf{n}^0(z) \cdot \mathbf{e}^{(2)}(\mathbf{k}_\perp^{(s)},\ z))^2 \\ & \times \sum_{j=1,2} \frac{\Big[\mathbf{e}_j(\mathbf{q}^{(sc)},\ z) \cdot \mathbf{e}^{(1)}(\mathbf{k}_\perp^{(i)},\ z)\Big]^2}{K_{jj} q^{(sc)2}(z) + (K_{33} - K_{jj})(\mathbf{q}^{(sc)}(z) \cdot \mathbf{n}^0(z))^2}. \end{aligned} \tag{6.11}$$

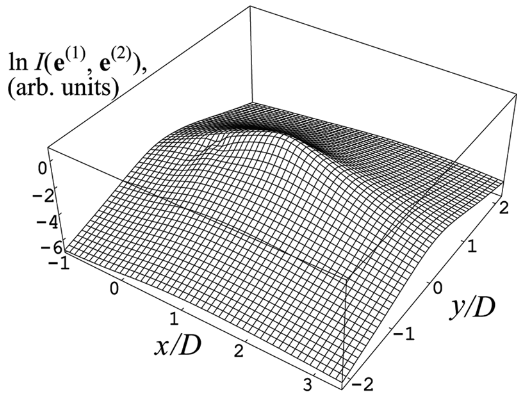
Intensity of the (e)–(o) scattering can be obtained from (o)–(e) scattering intensity if we substitute $\mathbf{e}^{(1)} \rightleftharpoons \mathbf{e}^{(2)}$ and $\mathbf{k}^{(s)} \rightleftharpoons \mathbf{k}^{(i)}$.

For (e)–(e) scattering both terms in brackets of Eq. (6.10) contribute in general to the intensity,

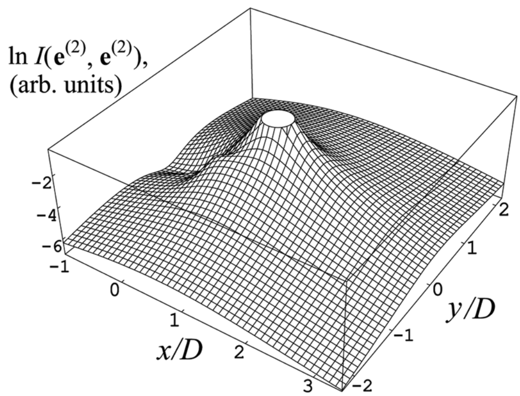
$$\begin{aligned} I(\mathbf{e}^{(2)}, \mathbf{e}^{(2)}) = {} & \frac{J_0 k_B T \varepsilon_a^2 |B^{(2)}(\mathbf{k}_\perp^{(s)}, L)|^2 E_0^{(2)2}}{|B^{(2)}(\mathbf{k}_\perp^{(i)}, 0)|^2} \left(\frac{z}{r}\right)^2 \frac{V_{sc}}{r^2 L} \\ & \times \int_0^L dz |B^{(2)}(\mathbf{k}_\perp^{(s)},\, z)|^2 |B^{(2)}(\mathbf{k}_\perp^{(i)},\, z)|^2 \\ & \times \sum_{j=1,2} \frac{1}{K_{jj} q^{(sc)2}(z) + (K_{33} - K_{jj})(\mathbf{q}^{(sc)}(z) \cdot \mathbf{n}^0(z))^2} \\ & \times \Big[(\mathbf{e}_j(\mathbf{q}^{(sc)},\, z) \cdot \mathbf{e}^{(2)}(\mathbf{k}_\perp^{(i)},\, z))(\mathbf{n}^0(z) \cdot \mathbf{e}^{(2)}(\mathbf{k}_\perp^{(s)},\, z)) \\ & + (\mathbf{e}_j(\mathbf{q}^{(sc)},\, z) \cdot \mathbf{e}^{(2)}(\mathbf{k}_\perp^{(s)},\, z))(\mathbf{n}^0(z) \cdot \mathbf{e}^{(2)}(\mathbf{k}_\perp^{(i)},\, z)) \Big]^2. \end{aligned} \quad (6.12)$$

The application of the WKB approximation imposes restrictions on the scattering geometries. First of all the scattering angle γ between vectors $\mathbf{k}_\perp^{(i)}$ and $\mathbf{k}_\perp^{(s)}$ is not small, ($\gamma \gg q_0/k_0 \sim \lambda/P$), since $q_\perp^{(sc)} P \gg 1$ in Eq. (6.10). Moreover the angles between the z axis and the wave vectors of the incident and scattered waves for the extraordinary beam can not be close to 90° due to the effect of the beam capture in the plane wave channel. At last there is a restriction on the thickness of CLC, $L \ll k_0/q_0^2 \sim \pi P^2/\lambda$, it is the consequence of the second inequality (6.9). From the latter inequality it follows that obtained equations are valid to the region of thickness from a rather thin CLC up to that containing many pitches.

We calculate the light scattering intensities $I(\mathbf{e}^{(1)}, \mathbf{e}^{(2)})$ and $I(\mathbf{e}^{(2)}, \mathbf{e}^{(2)})$ for the geometry shown in Figure 7. The results are represented as the intensity distribution on a flat screen, normal to the z axis. We choose the following CLC parameters: $\varepsilon_a = 1.0$, $\varepsilon_\perp = 2.5$, $K_{11} = 3.0 \cdot 10^{-6}$ dyn, $K_{22} = 2.0 \cdot 10^{-6}$ dyn, $K_{33} = 5.0 \cdot 10^{-6}$ dyn, the ratio of the CLC thickness to the pitch L/P is equal to $1/4$; the angle ϕ_i between the vector $\mathbf{k}_\perp^{(i)}$ and the vector director of the beam entering the CLC at $z = 0$ is set to be $\phi_i = \pi/4$. Figure 8 shows the intensity of the scattered light for the angle of incidence $\pi/4$ with respect to the z axis. For both types of scattering the intensity is maximal in the region of small scattering angles, $\theta_{sc} = \angle(\mathbf{k}^{(s)}, \mathbf{k}^{(i)}) \approx 0$. One can see that this region for (o)–(e) scattering is wider than for the (e)–(e) type. The intensity of (e)–(e) scattering for $\mathbf{k}^{(s)} \approx \mathbf{k}^{(i)}$ formally tends to infinity whereas for the (o)–(e) case it is finite. Here we do not consider the



(a)



(b)

FIGURE 8 The logarithm of the light scattering intensity (a) for (o)–(e) and (b) (e)–(e) types of scattering. All magnitudes are expressed in relative units. Coordinates x and y are measured in distances between the slab and the screen D.

region $\left|\mathbf{k}^{(s)} - \mathbf{k}^{(i)}\right| \lesssim q_0$ (the white spot in the Figure 8(b)), since our approach is not applicable in this region.

7. CONCLUSION

We have considered the problem of light propagation and scattering in cholesterics with the large pitch. In a helical medium the incident plane wave is transformed inside the scattering system into a normal wave of the helical medium which is not plane. The Green's function, i.e. the field of a point source, has a complex form as well. In particular, it depends on positions of the source and the receiver separately. Moreover, the Green's function has forbidden zones, i.e. regions where the wave can not penetrate. The trajectory and the polarization vector of the wave change in a rather complicated way. Besides, for certain directions the wave returns back and as a result a wave guide propagation takes place. The spatial correlation function of the permittivity tensor fluctuations resulting from the director fluctuations is not determined by the difference of the coordinates only, but essentially depends on their projections to the helical axis. In the present paper we have analyzed all these factors for CLC with the large pitch and have obtained the expressions for the light scattering intensity in a closed form.

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